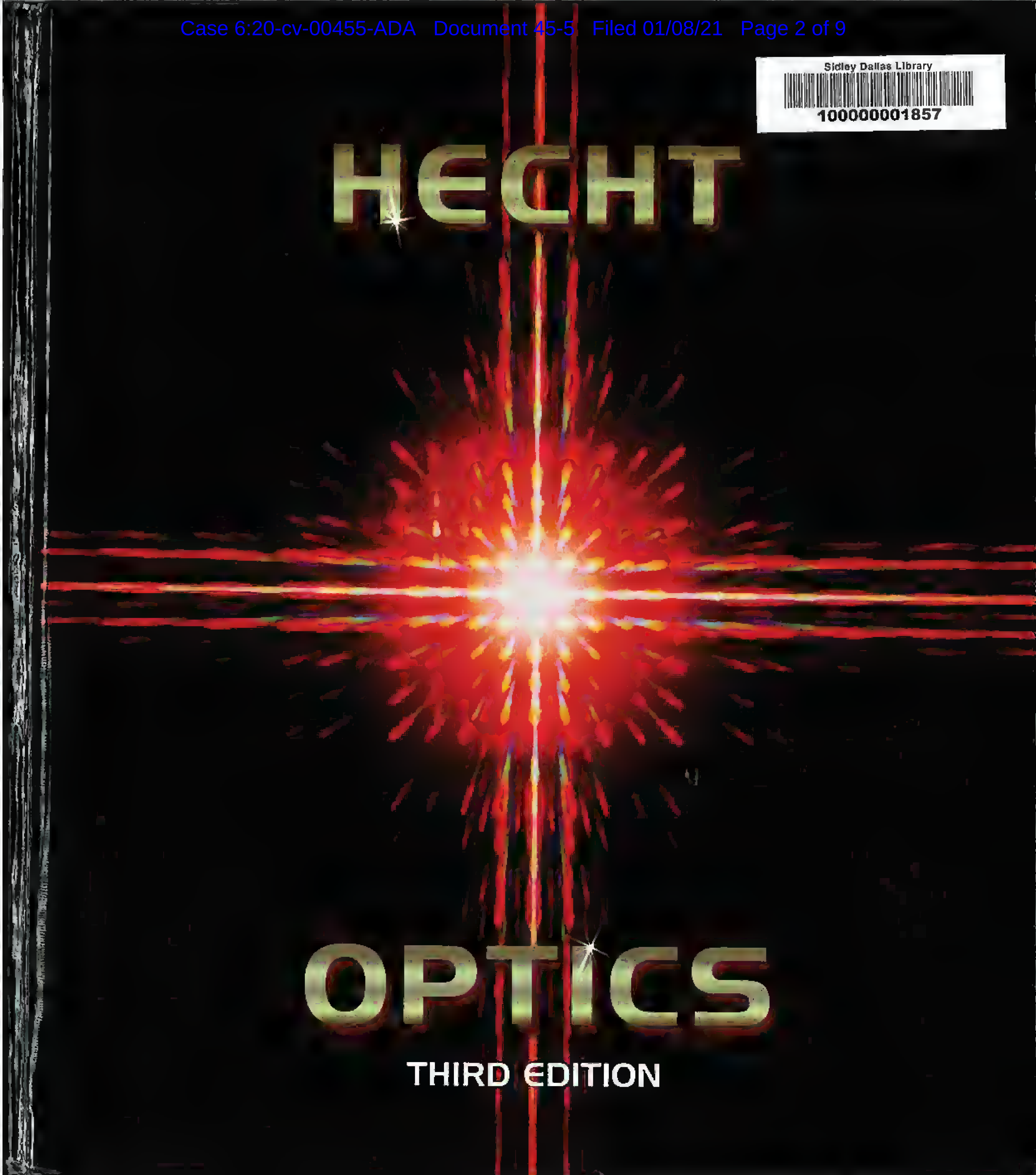
EXHIBIT E


HECHT
OPTICS
THIRD EDITION

OPTICS Third Edition

Eugene Hecht
Adelphi University

ADDISON-WESLEY
An imprint of Addison Wesley Longman, Inc.

Reading, Massachusetts • Menlo Park, California • New York • Harlow, England
Don Mills, Ontario • Sydney • Mexico City • Madrid • Amsterdam

Sponsoring Editor: *Julia Berrisford*
Production Supervisor: *Helen Wythe*
Manufacturing Supervisor: *Hugh Crawford*
Cover Designer: *Eileen R. Hoff*
Production Service: *HRS Electronic Text Management*
Text Design: *HRS Electronic Text Management*
Composition: *HRS Electronic Text Management*
Illustration: *Oxford Illustrators and HRS Electronic Text Management*

Library of Congress Cataloging-in-Publication Data

Hecht, Eugene
Optics / Eugene Hecht; — 3rd ed.
p. cm.
Includes index.
ISBN 0-201-83887-7
1. Optics. II. Title.
QC355.2.H42 1998
535—dc20

Printed in the United States.

3 4 5 6 7 8 9 0 - MA - 009998

where $m = 0, 1, 2, \ldots$, it will function as a half-wave plate ($\Delta\varphi = \pi, 3\pi, 5\pi$, etc.).

Although its behavior is simple to visualize, calcite is not often used to make retardation plates. It is brittle and difficult to handle in thin slices, but more than that, its birefringence, the difference between n_e and n_o, is a bit too large for convenience. On the other hand, quartz with its much smaller birefringence is frequently used, but it has no natural cleavage planes and must be cut, ground, and polished, making it rather expensive. The biaxial crystal mica is used most often. Several forms of mica serve the purpose admirably, for example, fluorophlogopite, biotite, or muscovite. The most commonly occurring variety is the pale brown muscovite. It is very easily cleaved into strong, flexible, and exceedingly thin large-area sections. Moreover, its two principal axes are almost exactly parallel to the cleavage planes. Along those axes the indices are about 1.599 and 1.594 for sodium light, and although these numbers vary slightly from one sample to the next, their difference is fairly constant. The minimum thickness of a mica half-wave plate is about 60 microns. Crystalline quartz, single crystal magnesium fluoride (for the IR range from 3000 nm to about 6000 nm), and cadmium sulfide (for the IR range from 6000 nm to about 12,000 nm) are also widely used for wave plates.

Retarders are also made from sheets of polyvinyl alcohol that have been stretched so as to align their long-chain organic molecules. Because of the evident anisotropy, electrons in the material do not experience the same binding forces along and perpendicular to the direction of these molecules. Substances of this sort are therefore permanently birefringent, even though they are not crystalline.

A rather nice half-wave plate can be made by just attaching a strip of old-fashioned glossy cellophane tape over the surface of a microscope slide. (Not all varieties work—the best is LePage's "Transparent Tape.") The fast axis, that is, the vibration direction of the faster of the two waves, corresponds to the transverse direction across the tape's width, and the slow axis is along its length. During its manufacture, cellophane (which is made from regenerated cellulose extracted from cotton or wood pulp) is formed into sheets, and in the process its molecules become aligned, leaving it birefringent. If you put your half-wave plate between crossed linear polarizers, it will show no effect when its principal axes coincide with those of the polarizers. If, however, it is set at 45° with respect to the polarizer, the E-field emerging from the tape will be flipped 90° and will be parallel to the transmission axis of the analyzer.

FIGURE 8.46 A hand holding a piece of Scotch tape stuck to a microscope slide between two crossed polaroids. (E.H.)

Light will pass through the region covered by the tape as if it were a hole cut in the black background of the crossed polarizers (Fig. 8.46). A piece of cellophane wrapping will generally also function as a half-wave plate. See if you can determine the orientation of each of its principal axes using the tape retarder and crossed Polaroids. (Notice the fine parallel ridges on the sheet cellophane.)

The Quarter-Wave Plate

The **quarter-wave plate** is an optical element that introduces a relative phase shift of $\Delta\varphi = \pi/2$ between the constituent orthogonal o- and e-components of a wave. It follows once again from Fig. 8.7 that a phase shift of 90° will convert linear to elliptical light and vice versa. It should be apparent that linear light incident parallel to either principal axis will be unaffected by any sort of retardation plate. You can't have a *relative* phase difference without having two components. With incident *natural* light, the two constituent $\mathscr{P}$-states are incoherent; that is, their relative phase difference changes randomly and rapidly. The introduction of an additional constant phase shift by any form of retarder will still result in a random phase difference and thus have no noticeable effect. When linear light at 45° to either principal axis is incident on a quarter-

wave plate, its o- and e-components have equal amplitudes. Under these special circumstances, a 90° phase shift converts the wave into circular light. Similarly, an incoming circular beam will emerge linearly polarized.

Quarter-wave plates are also usually made of quartz, mica, or organic polymeric plastic. In any case, the thickness of the birefringent material must satisfy the expression

$$d(|n_o - n_e|) = (4m + 1)\lambda_0/4$$

You can make a crude quarter-wave plate using household plastic food wrap, the thin stretchy stuff that comes on rolls. Like cellophane, it has ridges running in the long direction, which coincides with a principal axis. Overlap about a half dozen layers, being careful to keep the ridges parallel. Position the plastic at 45° to the axes of a polarizer and examine it through a rotating analyzer. Keep adding one layer at a time until the irradiance stays roughly constant as the analyzer turns; at that point you will have circular light and a quarter-wave plate. This is easier said than done in white light, but it's well worth trying.

Commercial wave plates are generally designated by their ***linear retardation***, which might be, for example, 140 nm for a quarter-wave plate. This simply means that the device has a 90° retardance only for green light of wavelength 560 nm (i.e., 4 × 140). The linear retardation is usually not given quite that precisely; 140 ± 20 nm is more realistic. The retardation of a wave plate can be increased or decreased from its specified value by tilting it somewhat. If the plate is rotated about its fast axis, the retardation will increase, whereas a rotation about the slow axis has the opposite effect. In this way a wave plate can be tuned to a specific frequency in a region about its nominal value.

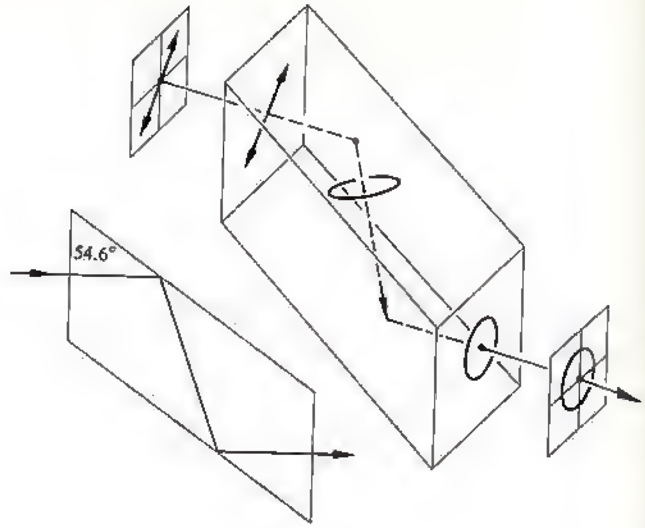


FIGURE 8.47 The Fresnel rhomb.

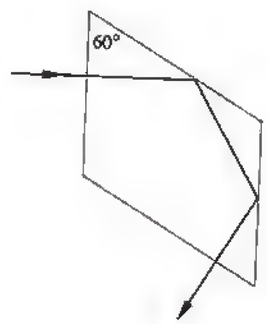


FIGURE 8.48 The Mooney rhomb.

The Fresnel Rhomb

We saw in Chapter 4 that the process of total internal reflection introduced a relative phase difference between the two orthogonal field components. The components parallel and perpendicular to the plane-of-incidence were shifted in phase with respect to each other. In glass ($n = 1.51$) a shift of 45° accompanies internal reflection at the particular incident angle of 54.6° (Fig. 4.43e). The Fresnel rhomb shown in Fig. 8.47 utilizes this effect by causing the beam to be internally reflected twice, thereby imparting a 90° relative phase shift to its components. If the incoming plane wave is linearly polarized at 45° to the plane-of-incidence, the field components $[E_i]_\parallel$ and $[E_i]_\perp$ will initially be equal. After the first reflection the wave within the glass will be elliptically polarized. After the second reflection it will be circular. Since the retardance is almost independent of frequency over a large range, the rhomb is essentially an *achromatic* 90° retarder. The Mooney rhomb ($n = 1.65$) shown in Fig. 8.48 is similar in principle, although its operating characteristics are different in some respects.

8.7.2 Compensators

A **compensator** *is an optical device that is capable of impressing a controllable retardance on a wave.* Unlike a wave plate where $\Delta\varphi$ is fixed, the relative phase difference arising from a